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Development of Self-Tuning Intelligent PID Controller Based on BPNN for Indoor Air Quality Control

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Abstract-For those who spend most of their time working indoors, the indoor air quality (IAQ) could affect their working efficiency and health. This paper presents an intelligent proportional-integral-derivative (PID) controller for IAQ control. Different from the traditional PID controller, this novel controller combined with Back-Propagation Neural Networks (BPNN) technology will regulate the PID parameters k_m k_b k_d automatically. In the present study, the algorithm of the BPNN-based PID controller is first discussed in details, and the control performance is then tested by simulation using MATLAB. The difficulty in IAQ control is the existence of control disturbance, time delay and measurement errors. The results show that the combined control algorithm has better performance on the systemic stability, disturbance resistance, fast response rate and small overshoot compared with traditional PID controller.

Keywords—Back-propagation, neural network, PID control, IAQ control, stability analysis.

I. INTRODUCTION

A growing number of heating, ventilating and air conditioning (HVAC) systems are being installed in buildings as a way of providing thermal comfort (TC) and improving indoor air quality (IAQ) for occupants [1]. Over the past decades, exposure to indoor air pollutants is believed to have increased due to a variety of factors, including the reduction of ventilation rates (for energy saving), the construction of more tightly sealed buildings, and the use of synthetic building materials and furnishings as well as chemically formulated personal care products, pesticides and household cleaners. As investigating all types of indoor air pollutants for general air quality monitoring and control is a complicated matter [2-4], it was suggested that the measurement and analysis of indoor carbon dioxide (CO2) concentration could be useful for understanding IAQ and ventilation effectiveness [5-7]. Although healthy people can tolerate a CO2 level up to 10,000 ppm without serious health effects, an acceptable indoor CO2 level should be kept below 1000 ppm or 650 ppm above the ambient level in order to prevent any accumulation of associated human body odour [8–9].

Being able to influence most of these factors, proper control strategies are needed to operate the building HVAC systems in order for optimizing occupant comfort and energy saving in built environment.

In order to achieve an optimal system performance in terms of response speed, systemic stability, disturbance resistance and small overshoot, a neural network proportional-integral-derivative (PID) controller with backpropagation based weight updating algorithm is proposed in this paper.

II. LITERATURE REVIEW

A. Current control methods

Different approaches for controlling indoor building environments have been developed for improving IAQ in buildings. They can be classified into two categories: (1) conventional methods; and (2) computational intelligence technologies.

Although intelligent control technologies are developing very fast in the recent decades, conventional control methods including dead zone control, PID control [10,11], optimal control [12–14], predictive control [15] and adaptive control [16], etc. are still widely used for indoor air quality control, especially PID controller because of its practicality.

Application of intelligent methods to the building control systems was started in the 1930s and artificial intelligence (AI) techniques were applied to the control of both conventional and bioclimatic buildings. Intelligent controllers were optimized by the use of evolutionary algorithms and developed for the control of subsystems of an intelligent building [17]. The synergy of the neural networks technology, with fuzzy logic, and evolutionary algorithms resulted in the so-called computational intelligence (CI), which has now been used in buildings. There are many computational intelligence control systems like Synergistic neuro-fuzzy techniques [18], Fuzzy systems [19], Neural network controllers [20], PI-like fuzzy controllers [21], etc. have been developed for IAQ control in buildings.



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B. Problems of current control methods

The introduced control methods are developed and employed in building HVAC systems to improve IAQ and some of them have excellent performance. However, the current control technologies still have problems need to be solved and their shortages can be summarized as follows:

- 1) Requiring bespoke design. Conventional control methods like PID control need to be designed based on the building model, and even for an adaptive controller, such knowledge is still required at certain extent. Therefore, it is quite likely that the mismatch of model could occur and poor control performance can be expected as a consequence.
- 2) Difficulty to put into applications. The weak points of fuzzy controllers are due to the difficulty of defining accurate membership functions and lack of the systematic procedure for the transformation of the expert knowledge into the rule base. Moreover, tuning parameters is a time consuming task. Neural network can automatic the process of tuning parameters, significantly reduce development time and results in a better performance. However, in neural nets, both knowledge extraction and knowledge representation are difficult. Other advanced control technologies also require expert knowledge library or need operators with specific skills, which prevents them from wide applications.
- 3) Lagging behind the development of HVAC technologies. Current control algorithms do not provide specific control for these HVAC systems and do not have excellent control performance.

C. Future perspectives

These shortages make it difficult to provide a quick solution to the demand for accurate control in building HVAC systems and to meet the demand for some new HVAC systems as well as to perform multi-task control with the aim to minimize energy consumption in buildings and maximize the improvement to the IEQ. In order to address the issues, the development of control technologies in the following areas is desirable:

- 1) New control approaches may merge both the conventional and advanced control methods. There are two possible ways: a method by which individual merits are combined, and another by which analogies between these are overlapped.
- 2) The control process that does not require intervention of the users with specific skills or knowledge.

3) Closed-loop, real-time and on-line learning ability. The IAQ performance evaluation system could be included to make a control process have the self-learning and modifying ability.

In order to achieve these goals, a new PID controller based on Back-Propagation Neural Networks (BPNN) learning algorithm will be discussed in next section.

III. DEVELOPMENT OF BPNN-BASED PID CONTROLLER

A. Control structure

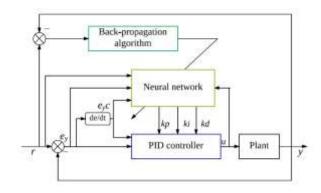


Figure 1: BPNN based PID control scheme

Figure 1 presents the structure of the proposed intelligent PID controller based on BPNN learning algorithm. It contains two parts: 1) a classic PID controller and 2) BPNN. The PID controller is used to control the control object (indoor air quality in building environment). The control performance depends on the setting of PID control parameters k_p , k_i and k_d which can be auto tuned by the BPNN. The BPNN uses an on-line training algorithm based on a gradient descent approach to update network weights and ensures that the designed neural network is able to calculate the desired PID control parameters for the PID controller. Therefore, in this control approach, by combining the classic PID control and the intelligent BPNN the targeted system output can be tracked with a guaranteed stability.

B. PID control algorithm

The incremental digital PID control algorithm can be expressed as follows:

$$u(k) = u(k-1) + k_p \{e_y(k) - e_y(k-1)\} + k_i e_y(k) + k_d \{e_y(k) - 2e_y(k-1) + e_y(k-2)\}$$
(1)



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Where u is the output of the PID controller, k_p is the proportional term, k_i is the integral term, k_d is the derivative term and e_y is the system error that can be expressed as follows:

$$e_{-}(k) = v(k) - r(k) \tag{2}$$

Where y is the system actual output and r is the system targeted output.

C. BPNN algorithm

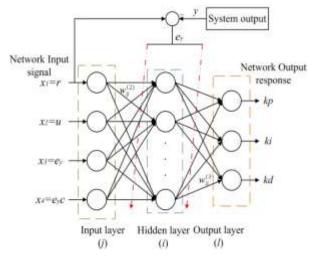


Figure 2: BPNN algorithm scheme

If the neural network has sufficient amount of neurons, it is able to approximate any continuous function with only one hidden layer [22–24]. Therefore, a neural network with only one hidden layer is designed. As shown in Figure 2, the proposed design is a four-input-three-output BPNN with three layers: input layer, one single hidden layer and output layer. In this section, the forward feed algorithm and the back-propagation weights adjustment rule is discussed in detail.

1 Input layer: The designed neural network has four inputs as shown in both Figure 1 and Figure 2 and they are:

$$\begin{cases} x_1 \\ x_2 \\ x_3 \\ x_4 \end{cases} = \begin{cases} r \\ u \\ e_y \\ e_y c \end{cases}$$

Where r, u and e_y is defined in equation (1) and (2); and e_yc is the changing rate of system error e_y that can be expressed as follows:

$$e_{v}c(k) = e_{v}(k) - e_{v}(k-1)$$
 (3)

Output of each neuron in the input layer is expressed as:

$$O_{:}^{(1)} = x(j)$$
 $\{j = 1, 2, 3, 4\}$ (4)

In the designed algorithm, superscript (1), (2) and (3) stands for input layer, hidden layer and output layer.

2 Hidden layer: Input of each neuron in the neural network hidden layer is expressed as follows:

$$in_i^{(2)}(k) = \sum_{j=1}^4 w_{ij}^{(2)} O_j^{(1)} \qquad \{i = 1, 2, \dots, N\}$$
 (5)

Where $w_{ij}^{(2)}$ is the weight connecting the input layer neurons to the hidden layer neurons and N is the number of neurons in the hidden layer.

Output of each neuron in the neural network hidden layer is expressed as follows:

$$O_i^{(2)}(k) = f(in_i^{(2)}(k))$$
 (6)

Where f(x) is the activation function in the hidden layer that presents the relation between the input and output of each neuron. Symmetrical Sigmoid function is used as the activation function and can be expressed as follows:

$$f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$
 (7)

3 Output layer: Input of each neuron in the output layer is given as:

$$in_l^{(3)}(k) = \sum_{i=1}^N w_{li}^{(3)} O_i^{(2)}(k) \qquad \{l = 1, 2, 3\}$$
 (8)

Where $w_{li}^{(3)}$ is the weight connecting the hidden layer neurons to the output layer neurons.

Output of each neuron in the output layer is given by:

$$O_l^{(3)}(k) = g(in_l^{(3)}(k))$$
 (9)

$$O_1^{(3)}(k) = k_p$$
 (10)

$$O_2^{(3)}(k) = k_i \tag{11}$$

$$O_3^{(3)}(k) = k_d (12)$$

Where g(x) is the activation function that presents the relation between the input and output of each neuron in the output layer. Outputs of the output layer are the PID parameters k_p , k_i and k_d .



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Since these values cannot be negative, the non-negative Sigmoid function is used as the activation function in output layer and it is given as:

$$g(x) = \frac{1}{2} (1 + \tanh(x)) = \frac{e^x}{e^x + e^{-x}}$$
 (13)

The proposed neural network can regulate the PID control parameters automatically and it can reduce sufficient time cost for engineers in control system design process. However, modelling errors often exist in model based process control and dramatically increase the difficulty to accurately control the process. Therefore, an on-line training algorithm is applied to adjust network weights for reducing the system error e_y in the design of the BPNN controller.

4 Weights update: In this algorithm, the system output error function is defined as:

$$E_{y}(k) = \frac{1}{2}(r(k) - y(k))^{2} = \frac{1}{2}e_{y}^{2}$$
(14)

The training process of the neural network model must be carried out before it can be put into use. This training process is repeated until the mean square error of the training data reaches the desired minimum. In the present work, the training process is based on back propagation. The basic idea of back propagation is to adjust the neuron weights using gradient descent algorithm on the error function in an iteration process. Generally, the adjustment of each weight from hidden-layer to output-layer can be expressed as follows:

$$\Delta w_{li}^{(3)}(k) = -\eta \frac{\partial E_{y}(k)}{\partial w_{li}^{(3)}}$$

$$\tag{15}$$

However, in order to avoid the 'local minima' which is the best known problem associated with back-propagation algorithm; a momentum term is added to the weight change in the proposed algorithm. This means that the weight change this iteration depends not just on the current error, but also on previous changes. So the adjustment of each weight from hidden-layer to output-layer is modified as follows:

$$\Delta w_{li}^{(3)}(k) = -\eta \frac{\partial E_{y}(k)}{\partial w_{li}^{(3)}} + \alpha \Delta w_{li}^{(3)}(k-1)$$

$$\tag{16}$$

Where is η learning rate, is α momentum factor.

$$\frac{\partial E_{y}(k)}{\partial w_{ij}^{(3)}(k)} = \frac{\partial E_{y}(k)}{\partial y(k)} \cdot \frac{\partial y(k)}{\partial u(k)} \cdot \frac{\partial u(k)}{\partial O_{i}^{(3)}(k)} \cdot \frac{\partial O_{i}^{(3)}(k)}{\partial in_{i}^{(3)}(k)} \cdot \frac{\partial in_{i}^{(3)}(k)}{\partial w_{ij}^{(3)}(k)}$$
(17)

$$\frac{\partial i n_l^{(3)}(k)}{\partial w_h^{(3)}(k)} = O_i^{(2)}(k) \tag{18}$$

Based on equation (1), (10), (11) and (12), the following equations are calculated:

$$\frac{\partial u(k)}{\partial O_1^3(k)} = e_y(k) - e_y(k-1) \tag{19}$$

$$\frac{\partial u(k)}{\partial O_2^{(3)}} = e_y(k) \tag{20}$$

$$\frac{\partial u(k)}{\partial O_3^{(3)}} = e_y(k) - 2e_y(k-1) + e_y(k-2) \tag{21}$$

Then, the learning algorithm of the weight update in output layer can be expressed as follows:

$$w_{li}^{(3)}(k+1) = w_{li}^{(3)}(k) + \Delta w_{li}^{(3)}(k)$$
(22)

$$\Delta w_{li}^{(3)}(k) = \alpha \Delta w_{li}^{(3)}(k-1) + \eta \delta_l^{(3)} O_i^{(2)}(k)$$
 (23)

$$\delta_l^{(3)} = e_y(k) \cdot \frac{\partial y(k)}{\partial u(k)} \cdot \frac{\partial u(k)}{\partial O_l^{(3)}(k)} \cdot g'(in_l^{(3)}(k))$$
 (24)

Where the first derivative of g(x) is given by:

$$g'(x) = g(x)(1 - g(x))$$
 (25)

And the learning algorithm of the weight update in hidden layer can be expressed as follows:

$$w_{ij}^{(2)}(k+1) = w_{ij}^{(2)}(k) + \Delta w_{ij}^{(2)}(k)$$
 (26)

$$\Delta w_{ii}^{(2)}(k) = \alpha \Delta w_{ii}^{(2)}(k-1) + \eta \delta_i^{(2)} O_i^{(1)}(k)$$
 (27)

$$\delta_i^{(2)} = f'(in_i^{(2)}(k)) \cdot \sum_{l=1}^3 \delta_l^{(3)} w_{li}^{(3)}(k)$$
 (28)

Where the first derivative of f(x) is given by:

$$f'(x) = \frac{(1 - f^2(x))}{2} \tag{29}$$

D. Summarise of BPNN based PID control

In the control process, the weights in the neural network are trained by the back-propagation weights adjustment rule in order to obtain the best PID parameters k_p , k_i and k_d for the PID controller.



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Therefore, an acceptable indoor air quality can be provided by the control of designed system. The algorithm of the BPNN based PID can be summarised as follows:

- 1) Initialize the each weight in the neural network $w_{ij}^{(2)}(k)$ and $w_{li}^{(3)}(k)$, as well as learning rate η and momentum factor α while k=1.
- 2) Collect data r(k) and y(k) and calculate the system error e_y using equation (2).
- 3) Calculate the input and output of each neuron and get the PID parameters k_p, k_i and k_d.
- 4) Calculate the output of PID controller using equation (1).
- 5) On-line training. Adjust the weight of each neuron in the neural network with the back-propagation learning algorithm in order to realize self-adaptive regulation of the PID parameters k_p, k_i and k_d.
- 6) Set k=k+1 and go back to rule 1).

IV. SIMULATION RESULTS AND DISCUSSION

The BPNN based PID control system is tested using MATLAB application tools. The difficulty in IAQ control for building environment is the existence of control disturbance and time delay. Simulations are conducted and results are analysed to discuss the performance including response speed, overshot, stability and adaptability of the proposed control strategy in IAQ control.

A time-varying system is used to represented IAQ control process for simulations to assess the effectiveness of the proposed neural PID control with on-line learning approach. As the process parameter becomes time variant, it causes difficulties to the system reference tracking control. The mathematical model of the control object is given as:

$$y(k) = \frac{a(k) \cdot y(k-1)}{1 + y^2(k-1)} + u(k-1) + 0.2u(k-2)$$

Where a(k) is a parameter varying over time and it is given as:

$$a(k)=1.4(1-0.9e^{-0.3k})$$

Set the initial learning rate $\eta = 0.28$ and momentum factor $\alpha = 0.04$. The weights in the neural network are initialized in the range [-0.5, 0.5] randomly.

Firstly, the step signal (r(k)=1) is introduced at time t=0. The simulation result of the proposed control system output is presented in Figure 3. As shown in the Figure, the system has very fast response speed and very small overshot.

Since there some disturbance factors in the actual control process, to further verify the robustness of the BPNN based PID control system, two disturbance loads at time t=0.8 and t=1.6 are added when the system is in the steady state. It can be seen in Figure 3 that when there are disturbances the control system can be fast stable and good stability can be provided.

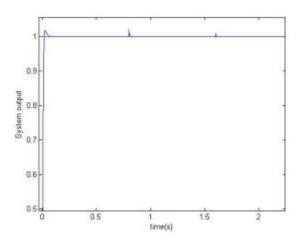


Figure 3: System output response to step input

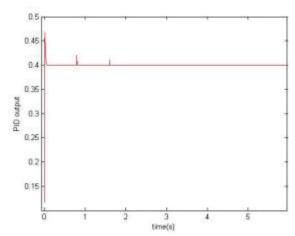


Figure 4: PID output response to step input

In Figure 4, PID controller output response to the step input signal and disturbances are presented and it performs the developed approach with small oscillations and short settling time.

The auto-tuning process of the PID parameters k_p , k_i and k_d can be observed in Figure 5. It shows that the proper PID parameters can be obtained using the online training algorithm based neural network control scheme.



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When the disturbances are suddenly added the neural network can quickly response to such situation and calculate the best k_p , k_i and k_d for the PID controller so that the system can be kept stable.

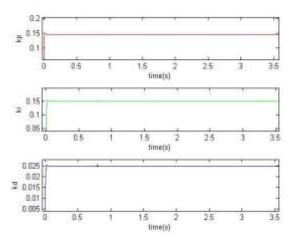


Figure 5: Auto-tuning of PID parameters (step input)

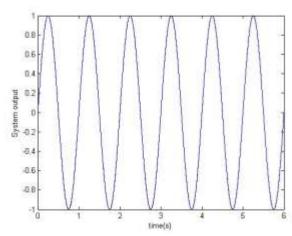


Figure 6 System output response to sine input

Then, to further verify the performance of the BPNN based intelligent PID control system a unit sine set-point $(r(k) = \sin(2 \pi t))$ is introduced at time t=0 and two disturbances at time t=0.8 and t=1.6. The initial weights in the developed neural network are selected randomly. These unsuitable weights may cause poor control performance; however, by adjusting them using the back-propagation learning algorithm the desired weights in the neural network can be fast obtained and are put to use for achieving the targeted system output.

In Figure 6, system output response to the sine input is presented. The result shows that the sine signal has been well tracked and the newly developed controller can cope with time variable parameter and time-varying system reference. There is no big overshot at the beginning that might be caused by the randomly set neural network weights. With the disturbances suddenly added, no oscillations were occurred and it can be observed more clearly in Figure 7 where describes the system error.

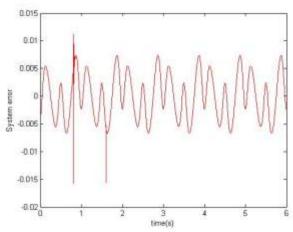


Figure 7: System error (sine input)

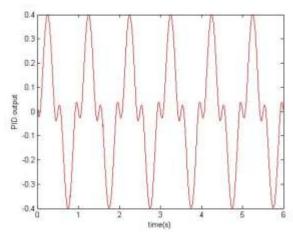


Figure 8: PID output response to sine input

In Figure 8, the result of PID controller output response to the sine input is presented ant it shows the PID control has fast response speed and stable steady state. This means that the PID parameters have been regulated accurately and the auto-tuning of k_p , k_i and k_d is shown in Figure 9.



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Figure 9 shows that the developed neural network control method has very good adaptability to parameter and system reference time variations. The desired output can be provided quickly in different situations.

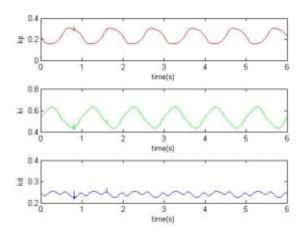


Figure 9: Auto-tuning of PID parameters (sine input)

As it has been shown with simulations that the back-propagation neural network technique is capable of providing suitable parameters for PID controller and the targeted system output can be achieved. Analysis shows that the response speed, stability, small system error and adaptability of the BPNN based PID control system have been guaranteed.

V. CONCLUSION AND FUTURE WORK

In this paper, an intelligent PID control system based on back-propagation neural network is introduced to address the drawback of PID control, i.e., it has to be designed based on the building model which is prone to the mismatch of model and poor control performance as a consequence and time consuming on parameters tuning. The proposed controller also avoids the need of knowledge extraction and knowledge representation in neural networks by a novel on-line training based neural network controller with respect to the variations observed from the system reference input and the actual output. Simulation trials have been conducted to show that the new developed control system presents excellent robustness to disturbance loads and process modeling errors and very good adaptability to cope with time variant process parameters, which was demonstrated from using the IAQ control model with timedependent parameter. In the future work, optimization approaches such as genetic algorithm will be used to optimize the neural network architecture to improve the control performance.

REFERENCES

- [1] Yu, B.F., Hu, Z.B. and Liu, M. 2009. Review of research on air conditioning systems and indoor air quality control for human health, Int J Refrig, (32), 3–20.
- [2] Yang, S., Wu, S., Yan, Y.Y., 2013. Control strategies for indoor environment quality and energy efficiency—a review, International Journal of Low-Carbon Technologies, doi:10.1093/ijlct/ctt051.
- [3] Hui, P.S., Wong, L.T. and Mui, K.W. 2006. Feasibility study of an express assessment protocol for the indoor air quality of airconditioned offices, Indoor and Built Environment, 15(4), 373–378.
- [4] Mui, K.W., Wong, L.T., Hui, P.S. and Law, K.Y. 2008. Epistemic evaluation of policy influence on workplace indoor air quality of Hong Kong in 1996–2005, Building Services Engineering Research and Technology, 29(2), 157–164.
- [5] Persily, A.K. 1997. Evaluating building IAQ and ventilation with indoor carbon dioxide, ASHRAE Transactions, 103(2), 193–203.
- [6] ASTM, 2003. Standard guide for using indoor carbon dioxide concentrations to evaluate indoor air quality and ventilation, D6245-98, (reapproved 2002).
- [7] Committee of European Normalization, Ventilation for buildings: design criteria for the indoor environment. Brussels, Luxembourg: Commission of the European Communities, Directorate General for Science, Research and Development Joint Research Centre-Environment Institute, CR1752, 1998.
- [8] ASHRAE, ANSI/ASHRAE Standard 62-2007, Design for acceptable indoor air quality, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., Atlanta, 2007.
- [9] Mui, K.W. and Wong, L.T. 2007. Evaluation of neutral criterion of indoor air quality for air-conditioned offices in subtropical climates, Building Services Engineering Research and Technology, 28(1), 23– 33.
- [10] Levermore, G.J. 1992. Building Energy Management Systems: An Application to Heating and Control. E & FN SPON.
- [11] Dounis, A.I., Bruant, M., Santamouris, M.J., et al. 1996. Comparison of conventional and fuzzy control of indoor air quality in buildings, J Intell Fuzzy Syst, (4), 131–40.
- [12] Wang, S. and Jin, X. 2000. Model-based optimal control of VAV air-conditioning system using genetic algorithms, Build Environ, (35),471–487.
- [13] Zaheer-uddin, M. and Zheng, G.R. 2000. Optimal control of time scheduled heating, ventilating and air conditioning processes in buildings, Energ Convers Manag, (41), 49–60.
- [14] Kummert M., Andre, P. and Nicolas, J. 2001. Optimal heating control in a passive solar commercial building, Solar Energ, (69), 103–116.
- [15] Milanic S.R.K. 1996. Neural network models for predictive control of a thermal plant. In: Proceedings of the International Conference on EANN.
- [16] Lam, H.N. 1993 Stochastic modeling and genetic algorithm based optimal control of air conditioning systems, Build Simulat, 435–441.
- [17] Sanchez, L., Doctor, F. and Hagras, H. 2004. An evolutionary algorithm for the off-line data driven generation of fuzzy controllers for intelligent buildings, In: Systems, Man and Cybernetics, IEEE International Conference, 42–47.
- [18] Kanarachos, A. and Geramanis, K. 1998. Multivariable control of single zone hydronic heating systems with neural networks, Energy Conversion Management, (13),1317-1336.



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- [19] Altrock, C., Arend, H.O., Krause, B., Steffens, C. and Behrens-Rommler, E. 1994. Adaptive fuzzy control applied to home heating system, Fuzzy Sets and Systems, (61), 29-35.
- [20] Liang, J. and Du, R. 2005. Thermal comfort control based on neural network for HVAC application. IEEE conference, 819-824.
- [21] Zhao, Z.Y., Tomizuka, M. and Isaka, S. 1993. Fuzzy gain scheduling of PID controllers, IEEE Transactions on Systems Man and Cybernetics, (123), 1392-1398.
- [22] Hornik, K., Stinchcombe, M. and White, H. 1989. Multilayer feedforward networks areuniversal approximators, Neural Networks 2(5), 359–366.
- [23] Cybenko, G. 1989. Approximation by superpositions of a sigmoidal function, Math.Control Signals Syst. (MCSS), (2), 303–314.
- [24] Ken-Ichi, F. 1989. On the approximate realization of continuous mappings by neuralnetworks, Neural Networks 2(3), 183–192.